DATA-DRIVEN PREDICTIVE MAINTENANCE USING THE GOMPERTZ CURVE

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Abstract
This paper presents the application of the Gompertz function in the context of machine condition monitoring and predictive maintenance. The proposed methodology focuses on modeling the nonlinear progression of wear using diagnostic signals collected during machine operation. While both structural and operational data can be utilized, operational signals such as temperature, pressure, and oil contamination provide the advantage of continuous, real time, and noninvasive monitoring. The asymptotic and asymmetric nature of the Gompertz function makes it particularly suitable for representing bounded degradation processes, especially in components with predefined wear thresholds. The paper introduces algorithms for estimating model parameters and calculating the remaining useful life (RUL) of machine elements. These algorithms, implemented in Python, enable real time evaluation of component condition and support data driven, proactive maintenance planning. The results demonstrate that Gompertz based modeling offers a robust and interpretable approach to improving machine reliability and optimizing maintenance strategies in accordance with Industry 4.0 principles.

Key words: *reliability, predictive maintenance, Gompertz curve, condition monitoring*

INTRODUCTION

Modern preventive diagnostic maintenance of machines is currently an increasingly addressed topic and part of modern industry, which allows us to detect emerging faults in time, especially using the values of diagnostic signals, and to implement measures that prevent them. *(Acernese, et al., 2020)* Examples of these operating parameters of the machine signals are vibrations, temperature, tribotechnical parameters of the oil or energy consumption witch are long-term monitored. *(Bouyahrouzi et al., 2022)* In real operation, however, it is often difficult to perform maintenance when the equipment requires it, as planned downtime must be taken into account. *(Esteban et al. 2022)*

When planning maintenance, there are three main approaches. The first is Corrective Maintenance, which is put on only when the fault manifests itself. The second is Preventive Maintenance witch schedules periodical maintenance actions based on operating hours. and the most up-to-date operation in relation to Industry 4.0 is predictive maintenance. *(Esteban et al. 2022, Proto et al. 2020)* Predictive maintenance is currently changing significantly as a result of Industry 4.0's improvements in industrial systems monitoring. It is now a data-driven field that frequently uses machine learning. *(Esteban et al. 2022, Aremu et al. 2020)*

In order to forecast potential equipment failure, predictive maintenance frequently uses reliability characteristics that are computed from historical data, such as reliability parameters or the findings of the Weibull analysis, which models time-to-failure and estimates the probability of failure over time. *(Petsinis et al. 2021, Liao et al. 2006)* Various statistical and analytical tools are used to combine this data with operational signals that are currently being monitored. *(Petsinis et al. 2021, Battifarano et al. 2018)* For example, logistic regression or logistic curve can be used to model the likelihood of failure based on the values of diagnostic variables, and regression and correlation analyses can be used to determine the relationships between operational parameters and failure risk. *(Battifarano et al. 2018, Liao et al. 2006)* This produces thorough prediction models that enable efficient maintenance planning and equipment operation optimization in addition to estimating the moment of failure. *(Liao et al. 2006, Yang et al. 2022)*

The logistic curve, or logistic regression, is a very widespread tool today, especially in the field of medicine, but its use in industry is growing rapidly. *(Hosmer et al. 2013, Bewick et al. 2005, Battifarano et al. 2018)* Its function in machine diagnostics is that when classifying states, whether a machine is in failure or without failure, it takes into account several diagnostic signals while simultaneously evaluating the risk of failure in the current state of the machine. *(Aremu et al. 2020, Petsinis et al. 2021)* The values that result are easy to interpret because they allow binary decision-making. The decision can be simple, for example, when we ask whether there is a current risk of a failure and the answer is either yes or no. *(Hosmer et al. 2013)* The result is values in the interval from 0 to 1, where values close to zero mean a low probability of failure and values close to one mean a very high probability of failure, and it therefore depends on the choice of a threshold value, above which a message is made that there is a risk of failure and it is necessary to plan an intervention. *(Liao et al. 2006, Hosmer et al. 2013, Yang et al. 2022)*

The aim of this study is to explore the application of the Gompertz curve in machine condition monitoring, with a focus on modeling the progression of wear using diagnostic signals. Both structural and operational data can be utilized for this purpose. In the following sections, the theoretical background of the Gompertz function, parameter estimation procedures, and the development of algorithms for real-time prediction and remaining useful life estimation will be described in detail.

MATERIALS AND METHODS

The Gompertz function is a nonlinear sigmoidal model suitable for describing gradual degradation processes in engineering systems. Its mathematical structure captures the behavior of many technical components where wear progresses slowly in the early stages, accelerates in the middle phase, and eventually stabilizes as the system approaches its physical limit or failure threshold. Figure 1 illustrates typical Gompertz curves for different values of the growth rate parameter b, demonstrating how the rate of degradation affects the curve's steepness and inflection point.

In the context of machine condition monitoring and predictive maintenance, the Gompertz curve is particularly useful for modeling bounded degradation phenomena based on operational diagnostic signals. These include temperature rise, pressure changes, or oil contamination levels—variables that typically evolve over time in a non-linear but saturating manner.

The general form of the Gompertz function *(Benzekry et al., 2014)* is expressed as:

$$y(t) = K \cdot e^{-e^{-b \cdot (t - t_0)}} \qquad (1)$$

where:

$y(t)$ is the value of the monitored diagnostic signal at operational time t

K is the upper asymptote representing the maximum value *(e.g., maximum allowable wear)*,

b is the growth rate,

t_0 is the inflection point where the growth is fastest.

In machine condition monitoring, particularly when assessing components such as hydraulic valves, sliders, and control elements, wear typically progresses in a nonlinear but bounded manner. The Gompertz function effectively captures this behavior, especially in systems with predefined operational thresholds or critical wear limits. Its asymptotic character reflects the physical constraints of such components.

Modern industrial systems, especially in robotic or semi-automated environments, often provide high availability of operational diagnostic data such as temperature, pressure, or contamination levels in hydraulic fluids. This enables non-invasive condition assessment based on real-time signals, eliminating the need for structural disassembly or direct inspection. To effectively apply the Gompertz function in technical diagnostics, it is crucial to accurately estimate its parameters K, b, and t_0. These parameters define the saturation level, growth rate, and inflection point of the Gompertz curve, respectively.

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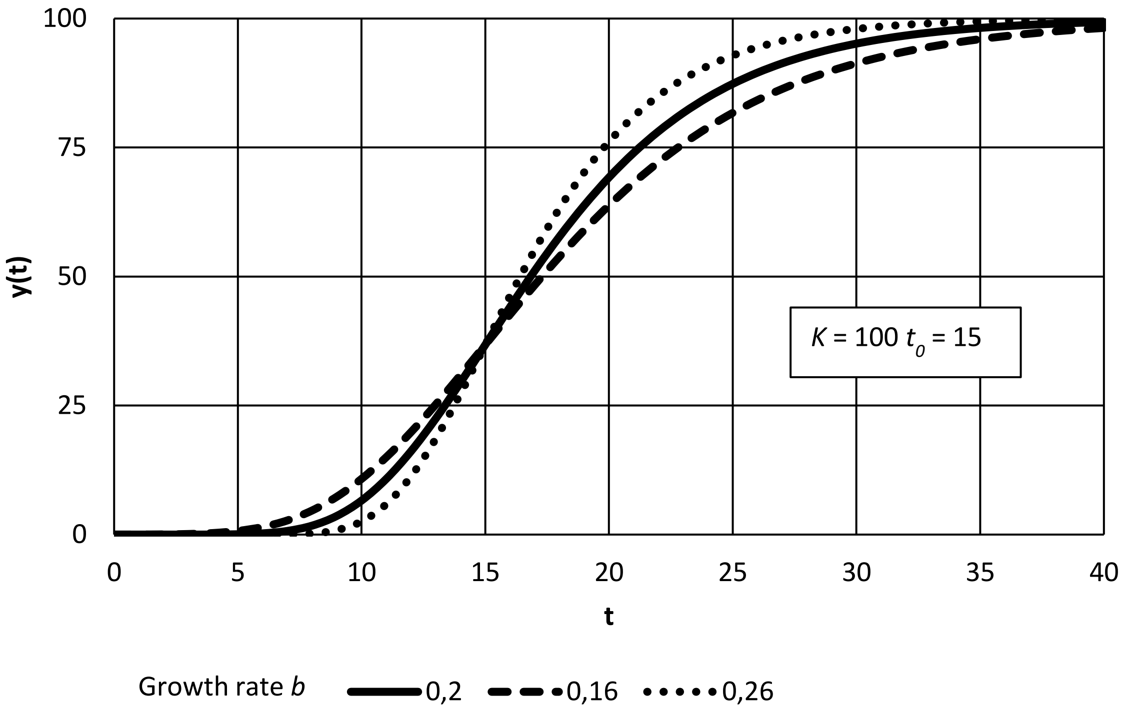

Fig. 1 Graph of Gompertz curve for three different parameters of the growth rate

Due to the nonlinear nature of the Gompertz function, linear regression methods are not applicable. Instead, nonlinear optimization must be used—specifically, methods that minimize the difference between measured data points and the model's predicted values.
The estimation problem is typically formulated as a minimization of the sum of squared residuals *(SSR)* *(Benzekry et al., 2014)*:

$$SSR = \sum_{i=1}^{n} \left(y_i - K \cdot e^{-e^{-b\cdot(t-t_0)}} \right)^2 \qquad (2)$$

Various programming environments such as Python, R, MATLAB, or even spreadsheet tools like Microsoft Excel can be used to estimate the parameters of the Gompertz function *(2)*. For practical and quick application, Excel offers a convenient platform that does not require any programming skills.
In Excel, the measured time-series data are entered alongside a column computing the Gompertz model output using initial parameter estimates. The squared differences between the measured and modeled values are calculated and summed to obtain the total error. Excel's Solver add-in can then be used to minimize this error by adjusting the parameters K, b, and t_0. A key advantage is that many machine components have defined upper wear thresholds, allowing the Gompertz curve's asymptotic parameter K to be either estimated or fixed according to technical specifications.

RESULTS AND DISCUSSION

To demonstrate the practical application of the Gompertz model in the context of machine condition monitoring, a computational tool was developed to automatically estimate the model parameters based on time-series data from operational diagnostic signals. These signals typically originate from sensors monitoring variables such as temperature, pressure, or oil contamination in hydraulic systems, and their evolution over time reflects the progressive degradation of machine components.
The goal of proposed algorithm is to fit the nonlinear Gompertz model to a set of observed measurements (t_i, S_i) and find optimal values for the model parameters K, b and t_0 by minimizing the sum of squared residuals *(SSR)*. The implementation enables both three-parameter fitting and simplified two-parameter fitting in cases where the maximum signal value K is known in advance.

For flexibility and reproducibility, the solution was implemented in Python using the *scipy.optimize* library, which is well-suited for nonlinear regression. The resulting program can be easily adapted for batch diagnostics or integration with online monitoring systems.

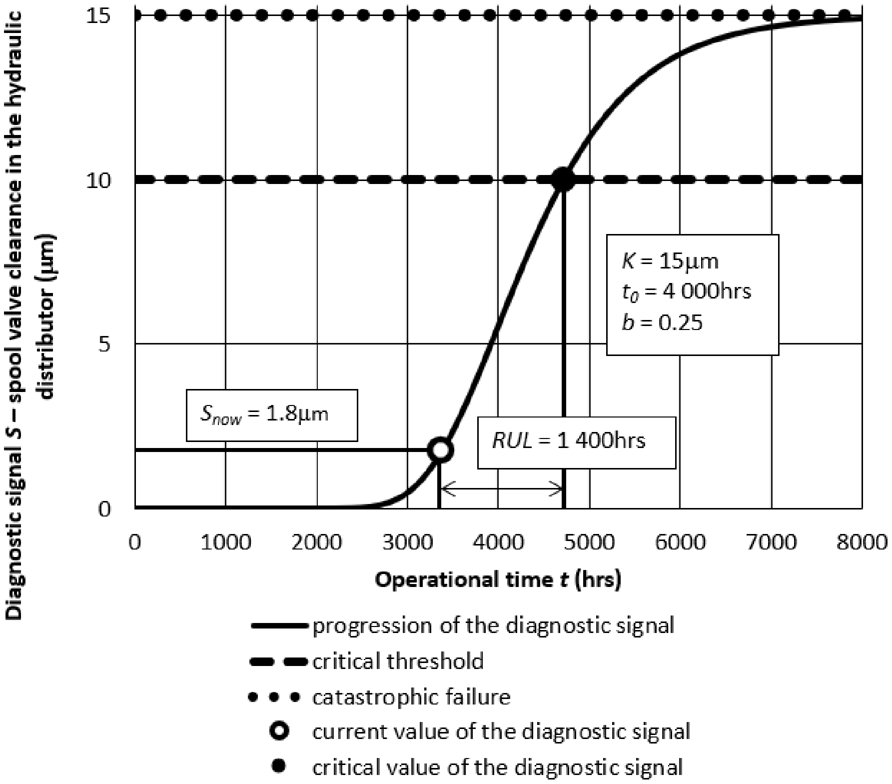

Fig. 2 Principle of remaining useful life *(RUL)* estimation using the Gompertz function

Once the Gompertz parameters *(Benzekry et al., 2014)* are estimated, the function can be used to predict the time at which the diagnostic signal will reach a predefined threshold—typically corresponding to a critical limit of wear, temperature, pressure, or other monitored variable. Let the critical threshold be denoted as S_{crit} *(3)*. The goal is to compute the future time S_{crit} such that:

$$S(t_{crit}) = y_{crit} = K \cdot exp(-exp(-b \cdot (t_{crit} - t_0))) \quad (3)$$

Solving for t_{crit} it is obtained inverse form:

$$t_{crit} = t_0 - \frac{1}{b} \cdot ln(-ln\frac{S_{crit}}{K})) \quad (4)$$

Remaining useful life *(RUL)* is then calculated as:

$$RUL = t_{crit} - t_{now} \quad (5)$$

where t_{now} is the time of the most recent measurement.

For the purposes of this study, a practical example was developed using empirical data obtained from real conditions. The monitored system is a hydraulic unit where the spool valve clearance in the hydraulic distributor represents a critical diagnostic parameter. The collected data were processed using the previously described methodology, and the parameters of the Gompertz function were estimated, as illustrated in Figure 2. The resulting curve was then used to estimate *RUL*. For instance, if the current value of the spool valve clearance is S_{now} =1.8 micrometers, the corresponding remaining time until the critical threshold is reached can be calculated using Equation *(5)*. In this case, the estimated *RUL* was approximately 1,400 operating hours.

Hydraulic systems present specific diagnostic challenges due to the interaction between mechanical components and pressurized fluid. In conventional approaches, the condition of critical components such as spool valves in hydraulic distributors are assessed by measuring the dimensions of functional surfaces. This enables the determination of structural parameters of diagnostic signals, such as the spool valve clearance. However, such measurements typically require system disassembly or offline diagnostic methods, which are time-consuming and not suitable for continuous monitoring. If a suitable diagnostic method based on real-time monitoring devices were available, it would be possible to determine these diagnostic signals using operational parameters instead of direct measurement. For hydraulic circuits, relevant operational signals may include the temperature of the hydraulic fluid, tribotechnical parameters, pressure variations within the system, or changes in flow behavior. These parameters can serve as indirect indicators of internal wear and degradation, including changes in spool valve clearance. The ability to derive diagnostic signals from operational data would enable truly non-invasive, continuous condition monitoring in hydraulic systems, aligning with the principles of predictive maintenance

Accurate estimation of *RUL* plays a crucial role in predictive maintenance and production management. It enables proactive decisions based on the actual condition of components rather than fixed service intervals. By predicting when a component will reach its wear threshold, production schedules can be adjusted to prevent unexpected downtime or even catastrophic failure. Maintenance can be better timed, spare parts prepared in advance, and repair operations efficiently organized. Moreover, knowledge of *RUL* supports strategic planning, including decisions on whether to invest in repair, replacement, or new equipment. In automated and interconnected systems, this predictive capability enhances reliability and overall operational performance. *(Green, E., & Taylor, L. 2024, Alenany, A., et al. 2023)* Similar methodologies have been successfully applied by *Wu, F. et al. (2023)* for hydraulic pumps where volumetric efficiency serves as a degradation indicator and *RUL* is predicted from the time-dependent trajectory. Also, *Kang et al. (2021)* demonstrated that machine-learning-based models can effectively predict *RUL* in scenarios where data are obtained from real-time operational sensors like temperature, pressure and flow.

CONCLUSIONS

This paper confirms the suitability of the Gompertz function for modeling degradation behavior in machine components within the framework of condition monitoring and predictive maintenance. Its asymmetric and bounded characteristics make it particularly appropriate for describing wear progression in components such as spool valves in hydraulic distributors, where predefined operational thresholds exist. The main contribution of this work is the development of algorithms for estimating the parameters of the Gompertz function based on diagnostic signals and calculating *RUL*. These algorithms, implemented in Python, were successfully applied to empirical data from a hydraulic system and enabled real time condition assessment of key components. By relying on operational diagnostic signals such as temperature, pressure, or oil contamination, the proposed approach allows for noninvasive and continuous monitoring. This enhances the practical implementation of predictive maintenance by shifting decision making from reactive or time based strategies toward condition based and data driven planning. The results demonstrate a clear potential to increase equipment reliability, reduce unplanned downtime, and optimize maintenance resources. Furthermore, this methodology aligns with the core principles of Industry 4.0, where intelligent monitoring, data integration, and algorithmic evaluation play a central role in improving operational performance across interconnected technical systems.

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