DETERMINATION OF OPERATIONAL PARAMETERS OF OLDER FRANCIS TURBINES – A REVIEW

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Abstract
For a qualified assessment of the reconstruction of an older Francis turbine in a small hydropower plant, it is necessary to identify individual operational parameters to ensure the highest possible efficiency of its future operation. This issue can be addressed using the physical similarity law of hydraulic machines. The paper describes the individual steps of the identification procedure and the subsequent determination of the annual electricity generation. The methodology is supplemented by a specific example of a small hydropower plant on the Úslava River. A comparison of the calculation results with actual measurements shows a good agreement in the basic turbine parameters, and the method can therefore be recommended for broader application.

Key words: *physical similarity law, efficiency, power, annual energy production*

INTRODUCTION
There are still many small hydropower plants (SHP) in the Czech Republic equipped with original, predominantly Francis turbines. Most of these units date back to the first half of the 20th century, the peak period of hydropower development in the country. In 1930, there were 15 638 hydropower plants, sawmills, hammers, and other facilities utilizing water power within the territory of the former Czechoslovak Republic *(Jirák, 1930)*. Despite adverse circumstances, a significant number of original Francis turbines have survived to this day, along with a growing interest among their current owners in restoring them to operation. While the turbine-generator sets themselves have withstood the passage of time, the situation with their technical documentation is less favourable. It has either survived only partially or, in most cases, has been irretrievably lost. As a result, future operators often lack answers to fundamental questions: What power output and how much energy will the reconstructed turbine be capable of producing?
The performance parameters of a small hydropower plant are determined both by site conditions (head, flow rate) and by the operational characteristics of the specific turbine, which are usually unknown. The aim of this paper is to describe a procedure for identifying the performance parameters of an older Francis turbine for which no technical documentation has survived, and to determine the expected annual electricity generation of the SHP. For better clarity, the procedure is illustrated with a specific example of an SHP on the Úslava River (SHP Úslava).

IDENTIFICATION OF FRANCIS TURBINE PARAMETERS
When designing water turbines, engineering practice applies the physical similarity law of hydraulic machines *(Mizra et al., 2024; Chen et al., 2022)*. The principles of this theory and its applications are described in *(Polák et al., 2016)*. This theory can also be used for reverse engineering purposes *(Ashish et al., 2020; Srdjan et al., 2018)* and for identifying the operational parameters of older Francis turbines. The following text presents general relationships and formulas for calculating operational parameters based on the dimensions of the runner. These formulas are subsequently applied to specific values for the SHP Úslava. The input parameters are two identifying dimensions of the runner (see Fig. 1) that characterize the Francis turbine:
- Width of the inlet channel of the runner, B_o
- Inlet diameter of the runner, D_1

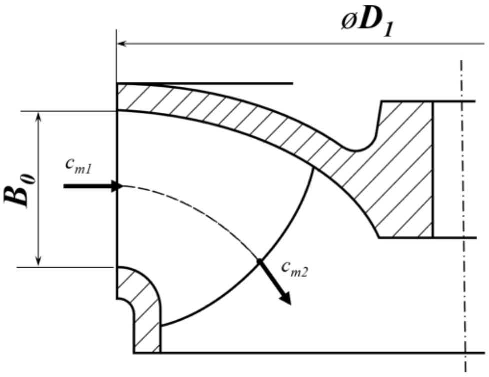

Fig. 1 Identification dimensions of the Francis turbine runner

The SHP Úslava is equipped with a turbine with dimensions: B_o = 0.200 m; D_1 = 0.650 m.
From these dimensions, we calculate the ratio:

$$x = \frac{B_o}{D_{1a}} = \frac{0.20}{0.65} = 0.308 \tag{1}$$

and substitute it into the equation for determining the specific speed n_s [min^{-1}]:

$$n_s = 11\ 014\ 600 \cdot x^6 - 13\ 338\ 500 \cdot x^5 + 6\ 471\ 600 \cdot x^4 - 1\ 596\ 100 \cdot x^3 + 209\ 700 \cdot x^2 - 12\ 839 \cdot x + 336 \tag{2}$$

Then for SHP Úslava:
$$n_s = 11\ 014\ 600 \cdot 0.31^6 - 13\ 338\ 500 \cdot 0.31^5 + 6\ 471\ 600 \cdot 0.31^4 - 1\ 596\ 100 \cdot 0.31^3 + 209\ 700 \cdot 0.31^2 - 12\ 839 \cdot 0.31 + 336 = 310\, min^{-1}$$

This equation (2) is a regression function (R^2 = 0.996) of data according to *(Hýbl, 1928)*. The specific speed defines basic turbine properties such as speed characteristics and efficiency. The value n_s = 310 min^{-1} corresponds to a so-called 'normal' Francis turbine.
From the specific speed, the expected efficiency can be calculated:

$$\eta_{T400} = 2{,}2 \cdot 10^{-9} \cdot n_s{}^3 - 3 \cdot 10^{-6} \cdot n_s{}^2 + 0{,}001\ 1 \cdot n_s + 0{,}71 \tag{3}$$

$$\eta_{T400} = 2{,}2 \cdot 10^{-9} \cdot 310^3 - 3 \cdot 10^{-6} \cdot 310^2 + 0{,}001\ 1 \cdot 310 + 0{,}71 = 0{,}83$$

which is again a regression function (R^2 = 0.905) of data according to *(Hýbl, 1928)*. The result indicates the efficiency measured in a laboratory on a turbine with an runner diameter D_1 = 400 mm. For a different runner diameter, the conversion formula (4) according to *(Hýbl, 1928)* applies where the diameter D_1 = 0.65 m is entered in meters:

$$\eta_T = 1 - (1 - \eta_{T400}) \frac{0.12 + \frac{0.021}{\sqrt{\frac{D_1}{4}}}}{0.12 + \frac{0.021}{\sqrt{\frac{0.4}{4}}}} \tag{4}$$

$$\eta_T = 1 - (1 - 0.83) \frac{0.12 + \frac{0.021}{\sqrt{\frac{0.65}{4}}}}{0.12 + \frac{0.021}{\sqrt{\frac{0.4}{4}}}} = 0.84$$

In SHP Úslava case, the turbine efficiency is 84%.

For further calculations, it is necessary to know the specific meridional velocity at the inlet to the runner c_{m1} [-] (see Fig.1), the equation of which (5) is again given by a regression function ($R^2 = 0.991$) of data according to *(Hýbl, 1928)*:

$$c_{m1} = 2.3 \cdot 10^{-9} \cdot {n_s}^3 - 2.25 \cdot 10^{-6} \cdot {n_s}^2 + 0.000\ 97 \cdot n_s + 0.076\ 7 \quad (5)$$

$$c_{m1} = 2.3 \cdot 10^{-9} \cdot 310^3 - 2.25 \cdot 10^{-6} \cdot 310^2 + 0.000\ 97 \cdot 310 + 0.076\ 7 = 0.230$$

Another important variable is the so-called "net head" of the turbine. This is determined by the difference between the geodetic head and the hydraulic head loss of a specific turbine. The geodetic head is determined at a given location by measuring the water levels upstream and downstream of the turbine. However, the hydraulic head loss of the turbine is usually unknown. The net head of the turbine can also be determined by calculation from equation (6) according to *(Melichar et al., 1998)*. However, this is only possible if the actual flow rate Q of the turbine is known from the original technical documentation. In the case of the SHP Úslava, this data was known from *(Jirák, 1930)*, where $Q = 0.65\ m^3 \cdot s^{-1}$ is stated. Then the net head is:

$$H = \frac{1}{2 \cdot g}\left(\frac{Q}{c_{m1} \cdot \pi \cdot D_1 \cdot B_o}\right)^2 \quad (6)$$

$$H = \frac{1}{2 \cdot 9.81}\left(\frac{0,65}{0.230 \cdot \pi \cdot 0.650 \cdot 0.200}\right)^2 = 2.45\ m$$

The geodetic head $H_G = 3.1$ m was determined by levelling and corresponds to the value according to *(Jirák, 1930)*. From this, we can determine the hydraulic head loss $H_L = H_G - H = 3.1 - 2.45 = 0.65$ m. If the actual turbine flow rate is unknown, the only option is to estimate the net head in a similar manner as in the example given here.

Therefore, if we know the net head, we can determine the turbine flow rate according to *(Melichar et al., 1998)* from the equation:

$$Q_n = c_{m1} \cdot \pi \cdot D_1 \cdot B_0 \sqrt{2 \cdot g \cdot H} \quad (7)$$

$$Q_n = 0.230 \cdot \pi \cdot 0.650 \cdot 0.200 \sqrt{2 \cdot 9.81 \cdot 2.45} = 0.65\ m^3 \cdot s^{-1}$$

However, in actual operation, the turbine will operate within a certain flow range determined by the current hydrological situation. This flow range must be considered when determining the total amount of energy produced in one year. The maximum flow through the turbine is determined by the flow rate:

$$Q_{max} = 1.20 \cdot Q_n \quad (8)$$

$$Q_{max} = 1.20 \cdot 0.65 = 0.78\ m^3 \cdot s^{-1}$$

The minimum flow rate at which the turbine is usually shut down because its operation is no longer economical corresponds to:

$$Q_{min} = 0.75 \cdot Q_n \quad (9)$$

$$Q_{max} = 0.75 \cdot 0.65 = 0.49\ m^3 \cdot s^{-1}$$

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HYDROLOGICAL PARAMETERS OF THE HYDROPOWER PLANT

To determine the available flow rate for the SHP, it is possible to either use paid hydrological data from the Czech Hydrometeorological Institute (CHMI) or perform in-situ measurements. For more details, see *(ČSN ISO 1438-1, 1998)* and *(ČSN EN ISO 748, 2008)*. The summarized flow rates can be presented in tables such as Table 1, which shows the flow rate expected over a given number of days per year.

Tab. 1: Hydrological data (CHI) – waterflow through the Úslava riverbed *(Hydrological data, 2010)*

Days	[d]	30	90	180	270	330	355	364
Waterflow, Úslava	$[m^3.s^{-1}]$	9.62	4.40	2.11	1.08	0.63	0.31	0.12

The table for the SHP Úslava indicates, for example, that the average flow rate will be at least 4.40 $m^3 \cdot s^{-1}$ for 90 days per year, or at least 0.63 $m^3 \cdot s^{-1}$ for 330 days per year or higher. However, it is necessary to deduct the minimum residual flow through the riverbed determined by the water authority pursuant to Section 36 of Act No. 254/2001 Coll. *(Česko, 2001)*. When water is drawn by the turbine, it ensures the minimum flow rate necessary to maintain its function in the natural environment throughout the entire original riverbed. Its value usually corresponds to the value of 355-day water. The resulting flow rate available to our small hydroelectric power plant is reduced here by Q_{355} = 0,31 $m^3 \cdot s^{-1}$. The final values of the maximum possible water intake for the turbine are shown in the third row of Table 2. This indicates that the plant can be operated for approximately 270 days per year.

GROSS ESTIMATE OF MAXIMUM ANNUAL ELECTRICITY PRODUCTION

Based on the calculated and measured values, the annual electricity production can be determined. The nominal turbine output is given by:

$$P = Q_n \cdot g \cdot H \cdot \eta_T \tag{10}$$

$$P = 0{,}65 \cdot g \cdot 2{,}45 \cdot 0{,}84 = 13{,}1\ kW$$

The instantaneous electrical output of the SHP at maximum turbine flow:

$$P_{max} = Q_{max} \cdot g \cdot H \cdot \eta_T \cdot \eta_P \cdot \eta_G \tag{11}$$

$$P_{max} = 0.78 \cdot 9.81 \cdot 2.45 \cdot 0.84 \cdot 0.95 \cdot 0.90 = 13.5\ kW_{el}$$

The instantaneous electrical output of the SHP at nominal turbine flow:

$$P_n = Q_n \cdot g \cdot H \cdot \eta_T \cdot \eta_P \cdot \eta_G \tag{12}$$

$$P_n = 0.65 \cdot 9.81 \cdot 2.45 \cdot 0.84 \cdot 0.95 \cdot 0.90 = 11.2\ kW_{el}$$

The instantaneous electrical output of the SHP at minimum turbine flow:

$$P_{min} = Q_{min} \cdot g \cdot H \cdot \eta_T \cdot \eta_P \cdot \eta_G \tag{13}$$

$$P_{min} = 0.49 \cdot 9.81 \cdot 2.45 \cdot 0.84 \cdot 0.95 \cdot 0.90 = 8.5\ kW_{el}$$

assuming: η_P … transfer efficiency (here a belt transmission: $\eta_P = 0.95$), η_G … generator efficiency (here asynchronous generator: $\eta_G = 0.90$)

Annual electricity production is derived from an overview of annual flows through the SHP. The number of operating hours and output at a given flow rate are specified for the number of days indicated. Their multiplication then determines the amount of energy produced in kWh for a given period. The total annual production is then their sum. The calculated values are clearly summarized in Table 2.

Tab. 2: Gross Estimate of Maximum Annual Electricity Production

Days	[d]	30	90	180	270	330	355	364
SHP flowrate	[$m^3 \cdot s^{-1}$]	9.31	4.09	1.8	0.77	0.32	0	0
Turbine flowrate	[$m^3 \cdot s^{-1}$]	0.79	0.79	0.79	0.66	0	0	0
Generator output	[kW]	13.5	13.5	13.5	11.2	0	0	0
Operating hours	[h]	180 · 24 = 4 320 h			2 160	0	0	0
Energy production	[kWh]	13.5 · 4 320 = 58 320 kWh			24 192	0	0	0
Annual energy production	[kWh]	82 512 kWh						

The total annual electricity production in the example of the SHP on the Úslava River will therefore be approximately 82 500 kWh.

TURBINE ROTATION SPEED

In order for the turbine to operate in optimal operating mode with maximum efficiency and performance, it is essential that its speed corresponds to the original design speed. Exceeding or falling below this speed always results in reduced efficiency and thus a loss of energy production.
First, it is necessary to know the specific circumferential speed u_{1s} [-] at diameter D_1, the equation of which (14) is given by the regression function ($R^2 = 0.999$) of data according to *(Hýbl, 1928)*:

$$u_{1s} = 0.0019 \cdot n_s + 0.46 \tag{14}$$

$$u_{1s} = 0.0019 \cdot 310 + 0.46 = 1.05$$

The turbine speed is then calculated from the circumferential speed of the runner:

$$n = 60 \cdot u_{1s} \cdot \frac{\sqrt{2 \cdot g \cdot H}}{\pi \cdot D_1} \tag{15}$$

$$n = 60 \cdot 1.05 \cdot \frac{\sqrt{2 \cdot 9.81 \cdot 2.45}}{\pi \cdot 0.65} = 214\, min^{-1}$$

RESULTS

The identification procedure determined that the SHP Úslava is equipped with a 'normal' Francis turbine which, at a flow rate $Q = 0.65\ m^3 \cdot s^{-1}$, net head $H = 2.45$ m, runner diameter $D_1 = 0.650$ m, and speed $n = 214\ min^{-1}$, achieves optimal efficiency of 84% and an nominal power output of $P = 13.1$ kW. With an average annual operation of 270 days, it produces approximately 82.5 MWh of electricity. Comparison of the calculated values with the preserved documentation parameters (head $H_G = 3.1$ m, flow rate $Q = 0.65\ m^3 \cdot s^{-1}$, nominal output $P = 15$ kW) shows good agreement between the identification method and reality.

CONCLUSIONS

This paper presents a methodology for determining the operational parameters of older Francis turbines for which technical documentation is missing. The calculations are based on the physical similarity law, using original design data from the period when these machines were manufactured. The example of the SHP Úslava demonstrates satisfactory agreement between the calculated basic operational parameters and actual values. The method also includes determination of annual electricity production of the SHP. Given the original design constraints, the method is reliably applicable to the range $0{,}06 \leq \frac{B_0}{D_{1a}} \leq 0{,}37$, $resp.$ $50 \leq n_s \leq 500$, in which the majority of original Francis turbines fall.

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